Common Types of Equations

Geologic relationships are often expressed as equations. These equations can take many forms:

- linear
- polynomial
- exponential
- logarithmic
- power

Linear Equations

linear equation have the form:

$$y = m \cdot x + b \tag{1}$$

m is the slope, b is the y intercept.

Polynomial Equations

- used to characterize more complicated relationships
- sum of powers, each multiplied by a constant. They have the form:

$$y = a = b \cdot x + c \cdot x^2 + d \cdot x^3 + e \cdot x^4 \dots \tag{2}$$

  - can include negative exponents
  - equation above is univariate, polynomials can also be multivariate equations.
- The highest power is the order of the polynomial (quadratic equations: second order polynomials.

Fractional Powers

- are 'roots' of numbers
- the square root is equal to a power of 0.5:

$$x^2 = y \tag{3}$$

$$(x^2)^{\frac{1}{2}} = y^{\frac{1}{2}} \tag{4}$$

$$x = y^{\frac{1}{2}} \tag{5}$$

Questions?

1. What is a root of an equation?
2. What is $x^{2.5}$? Express $x^{.7}$ as simple roots and powers.

Exponential Equations

- incorporate a variable as the exponent in the equation.

$$y = a \cdot b^x \tag{6}$$

- changes the behavior of the equation substantially.

Questions? (Using Pyplot/Pylab

1. What is the shape of this equation?
2. What about when 'b' is negative? (Explore variations)
3. What types of measurements can never go below zero?
4. What types of measurements are damped oscillations?

Basic plot with Matplotlib/Pylab

- import pylab as pl
  - imports extra math related stuff (eg. sin, cos, array)
- import matplotlib.pyplot as pl
- ipython –pylab
  - interactive plotting
  - won't 'lock' shell
- pl.plot
  - bivariate plot (line or scatter plot)
  - pass in one list or array: line plot
  - pass in two lists of equal size: scatter plot
  - option keywords to manipulate style (marker, linestyle, ...)

```
import matplotlib.pyplot as pl

# make a list for x over a reasonable range

x=[]
for i in range(0,101,1):#these have to be integers
    x.append(i)

# set constants
a=.5
b=-.9

# calculate values for y

y=[]
for i in x:
    y.append(a*b**i)

# create and show plot
pl.plot(x,y,marker='o',linestyle=':',color='red', markersize=1,markerfacecolor='blue')
pl.show()
```

Exponential Equations, e

- in many exponential equations, base of exponent is 'e'.
- irrational number equal to 2.718281828459
- described by the following relationships:

$$e = \lim_{x\to\infty}\left(1+\frac{1}{x}\right)^x \tag{7}$$

$$e = \sum_{x\to\infty}\left(\frac{1}{x!}\right) \tag{8}$$

- using 'e', the generalized exponential equation is:

$$y = a \cdot e^{b\cdot x} \tag{9}$$

Logarithms

Logarithmic equations are essentially the inverse of the exponential relationship. There are a host of logarithms, that depend on the base of the logarithm.

Using a base 2 log:

$$y = 2^x \tag{10}$$

$$log_2(y) = x \tag{11}$$

Two classes of logarithms appear frequently in Earth Sciences.

- the common logarithm (base 10)
- the natural logarithm (base e)
- 'log' typically refers to common logarithm
- 'ln' typically refers to natural logarithm
- some cases 'log' is the natural logarithm and log10 is the common logarithm
- verify behavior in software

Rules of logarithms parallel the rules of exponents:

$$10^a \cdot 10^b = 10^{a+b} \tag{12}$$

$$log(a \cdot b) = log(a) + log(b) \tag{13}$$

$$\frac{10^a}{10^b} = 10^{a-b} \tag{14}$$

$$log\left(\frac{a}{b}\right) = log(a) - log(b) \tag{15}$$

$$(10^a)^b = 10^{a \cdot b} \tag{16}$$

$$log(a^b) = b \cdot log(a) \tag{17}$$

To prove the rules of logarithm using the rules of exponents, consider the first relationship:

$$log(a \cdot b) = log(a) + log(b) \tag{18}$$

$$10^{log(a \cdot b)} = 10^{log(a)+log(b)} \tag{19}$$

$$a \cdot b = 10^{log(a)} \cdot 10^{log(b)} \tag{20}$$

$$a \cdot b = a \cdot b \tag{21}$$

Logarithms

- reduce multiplication/division to addition/subtraction - one reason logarithms were widely adopted
- used to 'compress' large ranges of numbers, extremes easier to view on graphs
- linearize non-linear relationships.

$$2123.4 \cdot 12546.7 = 10^{log(2123.4)} \cdot 10^{log(12546.7)} \tag{22}$$

$$10^{3.3270} \cdot 10^{4.0985} = 10^{7.4255} \tag{23}$$

$$10^{.4255} \cdot 10^7 = 2.66 \cdot 10^7 \tag{24}$$

Logarithmic scales are used throughout the earth sciences.

- Richter scale

- pH
- hydrograph

```
import matplotlib.pyplot as pl
'''
list comprehension
short-hand way to create a list of values
combines equations with for loop within list
'''

def eqn1(x):
    return 2.*2**x
def eqn2(x):
    return 2.*x**2

x=[.1*i for i in range(100)]
y1=[eqn1(i) for i in x]
y2=[eqn2(i) for i in x]
#make a semilog plot

pl.figure(1)
pl.semilogy(x,y1,label='eqn1')
pl.semilogy(x,y2,label='eqn2')
pl.grid(which='both')
pl.xlabel('x axis')
pl.ylabel('y axis')
pl.legend()

pl.figure(2)
pl.semilogx(x,y1,marker='o',markerfacecolor='pink',markeredgecolor='orange')
pl.semilogx(x,y2)
pl.grid(which='both')

pl.figure(3)
pl.loglog(x,y1)
pl.loglog(x,y2)
pl.grid(which='both')

pl.show()
```

```
import matplotlib.pyplot as pl
'''
subplots
Allow multiple plots on one figure
use three digit number (#rows,#cols,plot#)
'''

def eqn1(x):
    return 2.*2**x
def eqn2(x):
    return 2.*x**2

x=[.1*i for i in range(100)]
y1=[eqn1(i) for i in x]
y2=[eqn2(i) for i in x]

pl.subplot(2,1,1)
pl.semilogy(x,y1,label='eqn1')
pl.semilogy(x,y2,label='eqn2')
pl.grid(which='both')
pl.xlabel('x axis')
pl.ylabel('y axis')
pl.legend(loc='upper left')

pl.subplot(2,1,2)

pl.loglog(x,y1)
pl.loglog(x,y2)
pl.grid(which='both')

pl.show()
```


Geometric and Arithmatic Series

- Geometric and arithmetic progressions are common patterns in Earth Sciences.
- breakdown of rock to finer grains follow a geometric progression. 16,8,4,2,1,.5,...
- progression basis for the ϕ grain-size scale, each division between grain-size classes is two raised to some whole number power. Sand has grain size between -4 (2^{-4} mm) and 1(2^{1} mm) in the ϕ scale.

Progressions

- Geometric progression: sequence where next number is the previous number multiplied by a constant (the common ratio).
- Arithmetic progressions: refer to a sequence where next number is the sum of the previous number and a constant (the common difference).
- Current human population is growing geometrically...as time progresses arithmetically, human population increases by some factor.

Human population growth is commonly described as doubling every 40 years. In 2000, the world population reached 6 billion people. What will the world population be in 20 years?

Logarithms are based on the relationship between an arithmetic and geometric progression.
Writing out a geometric and arithmetic progression: 0, 1 1, 2 2, 4 3, 8 4, 16 5, 32 6, 64 7, 128 8, 256 9, 512
There is a clear relationship between each column.

$$C2 = 2^{C1} \tag{25}$$

This relationship can be turned around, giving the logarithm.

$$\log_2(C2) = C1 \tag{26}$$

Power Functions

Power functions have the form:

$$y = a \cdot x^b$$

- relationship common throughout nature.
- It is present if geomorphology (stream length to wathershed area), hydrogeology (distribution of fractures), geophysics (earthquake distribution), etc
- What does the graph of a power law and exponential relationship look like?

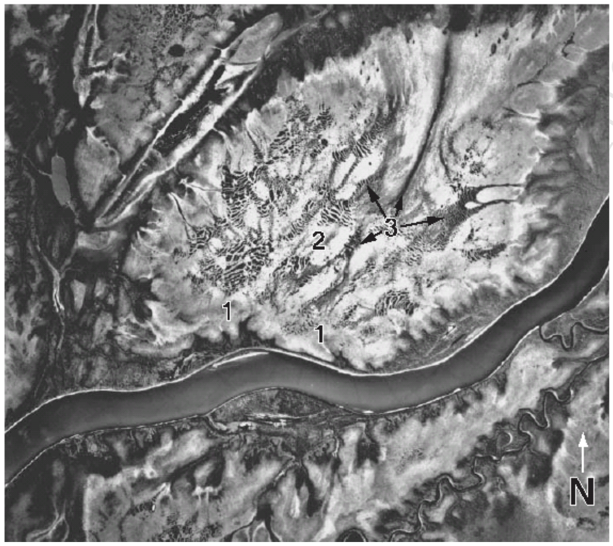
Using Equations

Solving Problems

- problems are not typically initially cast with equations.
- posed as word problems that must be placed into a mathematical framework.
- steps (CG6) for solving a word problem are:
  - understand the problem
  - develop a plan
  - carry out the plan
  - look back

Examples

- Power law relationships describe morphology of peat landforms.
- Glaser (1987) found a power law relationship between the area of bog islands, their length, and their width.
- relationship used to infer that hydrologic processes were acting on the islands to shape them.
- a similar power law relationship exists across many different peatland systems suggests processes responsible for shaping the bog islands is universal.



Geochemical equations

- commonly solved by casting them into 'log-space'.
- CG2: using logarithms simplifies geochemical calculations.
- graphs plotted using logarithmic scales to show the behavior of solutes under different geochemical conditions.
- common geochemical plot is log(C)-pH graph; a plot of pH verses the concentration of various solutes.

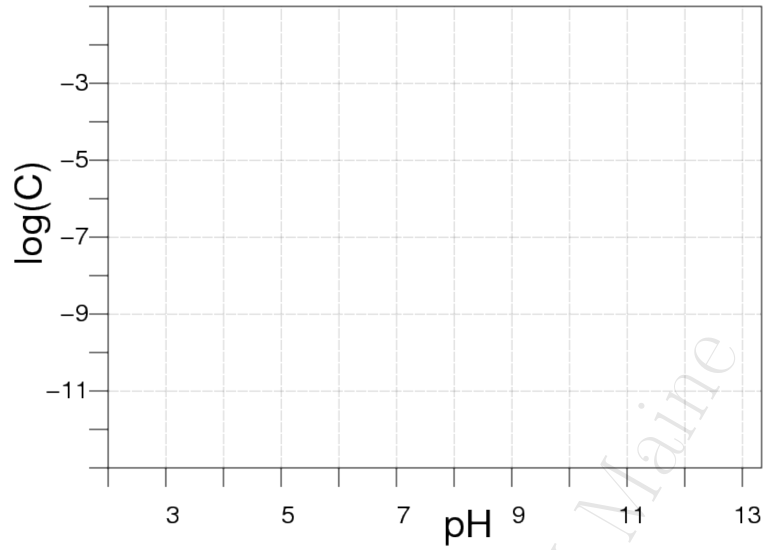
Graph the following relationships:

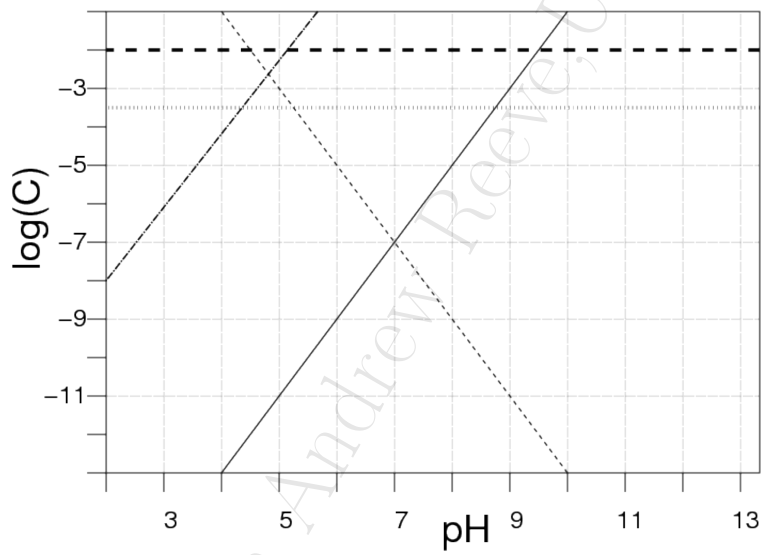
$$H_2O = H^+ + OH^- \, ; \, K_{eq} = 10^{-14} \tag{27}$$
$$CO_2 + H_2O = H_2CO_3 \, ; \, K_{eq} = 10^{-1.5} \tag{28}$$
$$H_2CO_3 = H^+ + HCO_3^- \, ; \, K_{eq} = 10^{-6.3} \tag{29}$$

Assuming $PCO_2 = 10^{-2}$

- Understand the problem?
- Develop Plan...cast chemical reactions into equilibrium equations
- Casts these equations into linear form that can be graphed.

$$\frac{(H^+)(OH^-)}{(H_2O)} = 10^{-14} \tag{30}$$
$$log(\frac{(H^+)(OH^-)}{(H_2O)}) = log(10^{-14}) \tag{31}$$
$$log(H^+) + log(OH^-) - log(H_2O) = -14 \tag{32}$$
$$-pH + log(OH^-) = -14 \tag{33}$$
$$log(OH^-) = 1(pH) + (-14) \tag{34}$$


log(C)
−3
−5
−7
−9
−11
3
5
7
pH
9
11
13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