What Controls Rate of Groundwater Flow?

- Hydraulic Gradient
- Hydraulic Conductivity

Hydraulic Gradients

- Drive groundwater flow
- Flow from high to low hydraulic head
- Hydraulic gradients are *gradients*

$$I = \nabla h = \frac{\partial h}{\partial x} i + \frac{\partial h}{\partial y} j + \frac{\partial h}{\partial z} k$$

- 1-D problems for now

$$I = \frac{dh}{dx}$$

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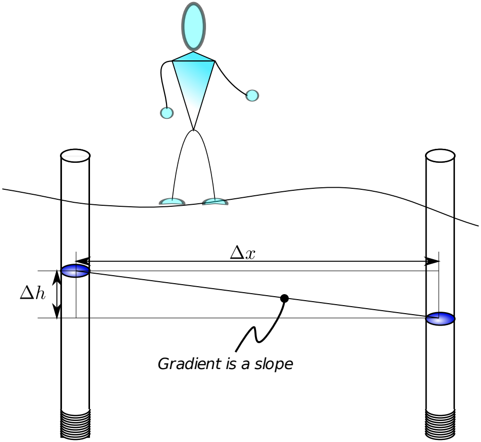
Hydraulic Gradient



Hydraulic Conductivity (K)

- measure of ease that a fluid moves through porous media
- media properties: permeability (related to pore size and open cross-sectional area)
- fluid properties: density and viscosity

$$K = \frac{\kappa \rho g}{\mu}$$

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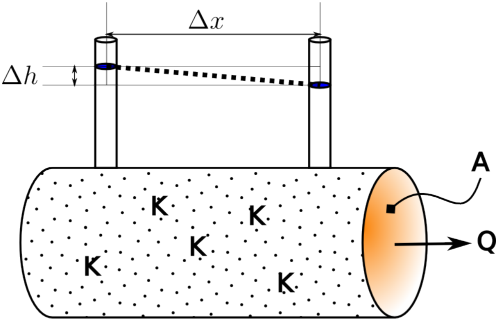
Hydraulic Conductivity (K)

Values for K (ft/day) (Halford and Kuniansky, 2002, USGS Open-File Report 02-197)

Aquifer Material	Extreme Minimum	Likely Minimum	Likely Maximum	Extreme Maximum
Gravel	90	300	3000	3000
Sand and Gravel Mixes	1	30	300	300
Coarse Sand	50	70	300	300
Medium Sand	1	20	70	200
Fine Sand	0.05	3	20	20
Silt, Loess	0.0003	0.001	0.1	6
Till	3.00E-07	0.003	0.3	0.6
Unweathered Marine Clay	2.00E-07	2.00E-07	0.0006	0.0006
Limestone, Dolomite	0.0003	0.004	0.1	2
Permeable Basalt	0.1	1	100	6000
Basalt	0	0.03	0.1	0.1
Fractured Igneous and Metamorphic Rock	0.001	0.05	10	100

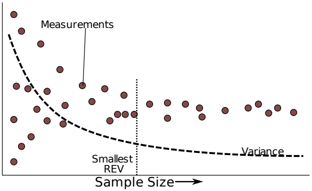
Darcy's Law



$$Q = -K \cdot A \frac{dh}{dx}$$

Limits of Darcy's law

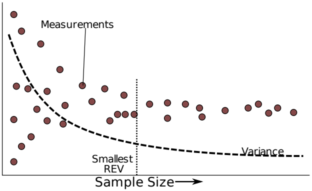
- Sample scale, REV
- laminar flow (slow velocity)



$$R_e = \frac{v \cdot d \cdot \rho}{\mu}$$

Limits of Darcy's law

- Sample scale, REV
- laminar flow (slow velocity)
  - increasing turbulance, friction loss
  - R_e below 1 for laminar conditions
  - $R_e > 10$ indicates turbulances
  - R_e between 1 and 10, transitional zone



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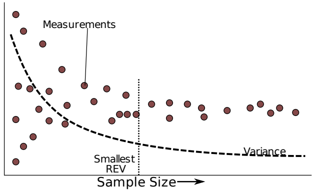

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Variations on Darcy's Law

- Specific Discharge or Darcy Velocity (q)
  - Discharge per unit area
  - units of velocity?
- Seepage Velocity or Avg. Linear Vel. (v)

$$q = -K \frac{dh}{dl} \qquad\qquad v = -\frac{K}{n_e} \frac{dh}{dl}$$

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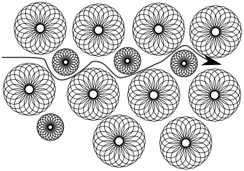

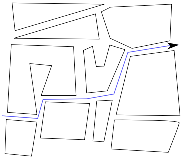

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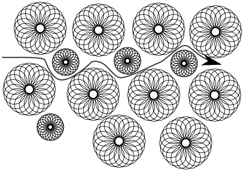

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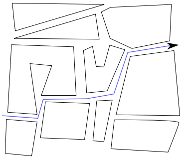

Darcy's Law

Question?

Why is there a minus sign in Darcy's Law?

Answer

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- To reverse sign on hydraulic gradient. Positive discharge then indicates flow to right.
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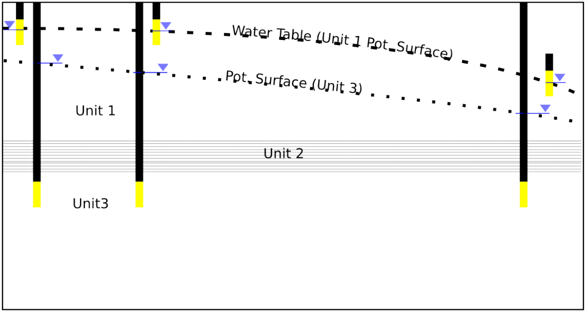
Potentiometric Surface

- Surface representing Hydraulic ‘energy’ (head)
  - Across a single geologic unit
  - or based on other constraint
- Water Table (surface with $\Psi = 0$)



2-D Flow Direction from Head Data

- Darcy's Law in 2-D
  - hydraulic gradient and discharge have components
  - vector math!
- What is minimum number of wells that define a plane (potentiometric surface)?

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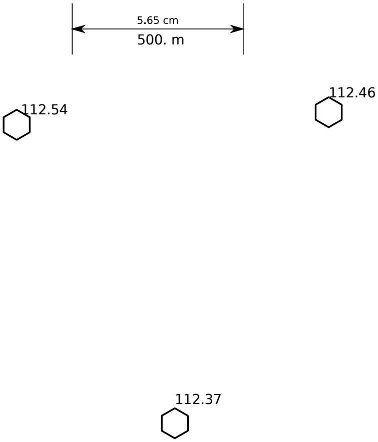

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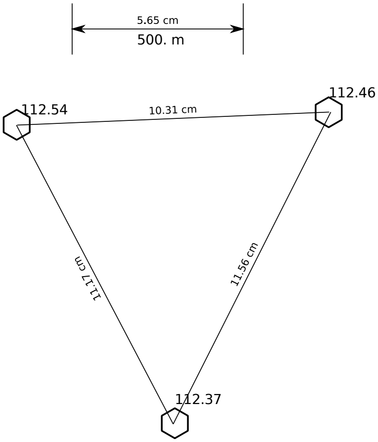

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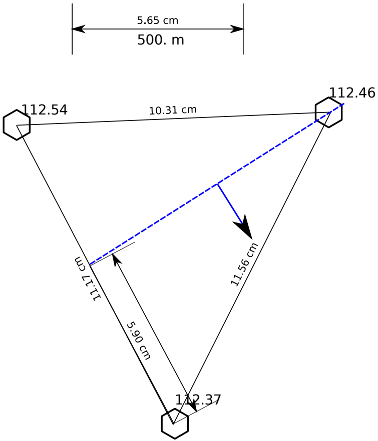
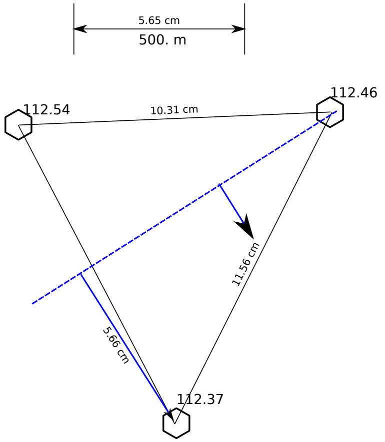


2-D Flow Direction from Head Data

Question?

What is the magnitude of the hydraulic Gradient?

2-D Flow Direction from Head Data



$$\Delta l = 5.66cm \cdot \frac{500m}{5.65cm}$$

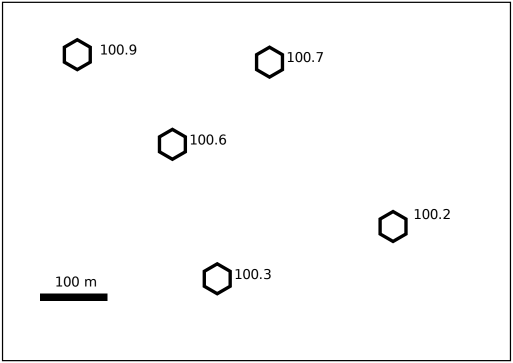
$$\Delta l = 500.9m$$

$$\Delta h = 112.46 - 112.37$$

$$\Delta h = 0.09$$

$$\frac{\Delta h}{\Delta l} = 0.000180 = 1.80 \cdot 10^{-4}$$

Contouring



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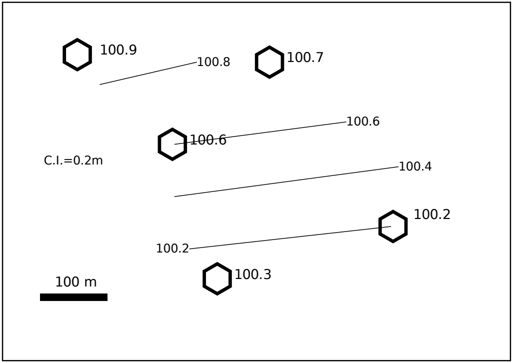


Computer-Based Interpolation

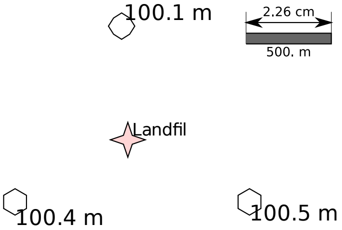
- Kriging, Splines, etc.
- Useful methods, but need to understand methodology and review results.

A computer lets you make more mistakes faster than any other human invention in human history ... with the possible exception of handguns and tequila.
Mitch Ratcliffe

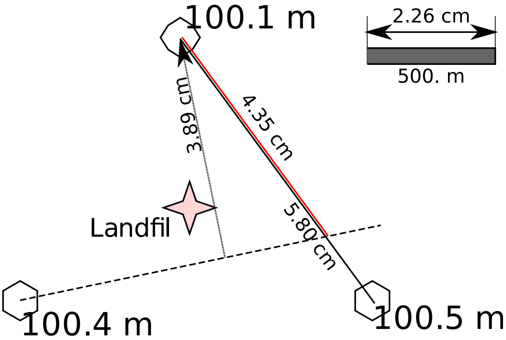
Simple Groundwater Flow

Question?

A landfill is located over a 10 m thick fine sand aquifer and monitoring wells are installed around it. Determine the direction of groundwater flow. The property lines of adjacent lots are 500 to 700 m from the landfill. How long will it take leachate carried by the groundwater to cross a property line? What information do you need to 'make up' to answer this question?



Simple Groundwater Flow



Simple Groundwater Flow

Question?

A large earthen dam is constructed to impound a river. Water levels in either side of the dam are 10 m and 5 m above a horizontal and impermeable clay. The dam is 100m wide, 10m thick, 12m high, and is made from material with K=$1. \cdot 10^{-6}\ m \cdot sec^{-1}$. How much water is flowing through the dam every day?

Simple Groundwater Flow

Answer-The Hard Way

Note that the cross sectional area and gradient will vary through the dam. Set up equation that incorporates this change.

$$Q = -KA\frac{dh}{dx} = -K(100 \cdot (5 + 0.5 \cdot x))\frac{dh}{dx}$$

$$\frac{-100 \cdot K}{Q}dh = \frac{dx}{5 + 0.5 \cdot x}$$

$$u = (5 + 0.5 \cdot x); du = 0.5dx$$

$$\int_h \frac{-100 \cdot K}{Q}dh = \int_x \frac{2 \cdot du}{u}$$

$$\frac{-100 \cdot K}{Q}(10 - 5) = 2\ln(u)$$

$$\frac{-100 \cdot K}{Q} \cdot h\Big|_5^{10} = 2\left(\ln(5 + 0.5 \cdot x)\right)\Big|_0^{10}$$

$$Q = 3.607 \cdot 10^{-4}\ m \cdot sec^{-3}$$

Simple Groundwater Flow

Answer-The Simple (but not quite right) Way

Note that the cross sectional area and gradient will vary through the dam and estimate an 'average' value.

$$Q = -KA\frac{dh}{dx}$$

$$\frac{dh}{dx} \approx \frac{5}{10}$$

$$A \approx 100\frac{10+5}{2}$$

$$Q = -K \cdot 750\frac{1}{2}$$

$$Q = 3.75 \cdot 10^{-4}\ m \cdot sec^{-3}$$

Flow in Fracture Networks

- Discrete Fracture Flow
- Continuum Approach
- Fractures are Difficult to Characterize

Flow Between Plates

- Cubic Law
- aperture thickness dominates equation (cubed value)
- assumes smooth walls
- difficult to measure and spatially variable
- b changes with water pressure

$$Q = -\frac{b^3 \cdot \rho \cdot g}{12 \cdot \mu} w \frac{dh}{dl}$$

b is aperture thickness; w is width along fracture

$$K = \frac{b^3 \cdot \rho \cdot g}{12 \cdot \mu} N$$

N is # of fractures per unit width

Fractures Flow and Continuum Approach

- Large REV
- Difficult to determine appropriate scale
- Percolation Theory

Unsat. Flow and Richard's Equation

- Unsaturated zone
- Hydraulic cond. dependent on water content
- Negative pressure head in unsaturated materials

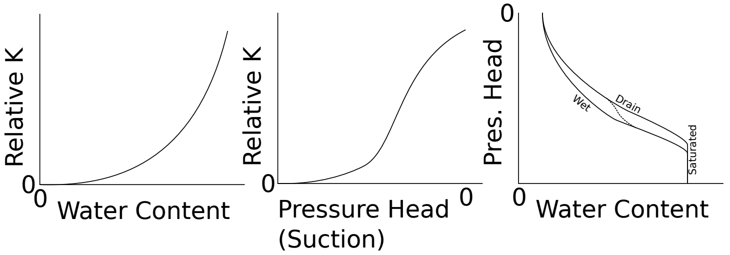
For vertical direction (eg. infiltration):

$$q = -K(\Theta)\frac{dh}{dz}$$

$$q = -K(\Theta)\frac{d(\Psi + z)}{dz}$$

Unsat. Hydraulic Cond.

- $K(\Theta) < K$, pore spaces blocked by air
- $K(\Theta)$ decreases with Θ and Ψ
- Usually displayed as relative hydraulic cond. $\left(\frac{K(\Theta)}{K}\right)$
- Hysteresis (water content vs pressure head)



Mathematical Relationships

Brooks and Corey

$$\frac{K(\Theta)}{K} = \left(\frac{\Theta - \Theta_{resid}}{n - \Theta_{resid}}\right)^{a}$$

a is function of pore size, equation applicable when pressure below bubbling pressure (pressure/suction needed to force water out of pores)

van Genuchten

$$\frac{K(\Theta)}{K} =$$

$$\left(\frac{\Theta - \Theta_{resid}}{n - \Theta_{resid}}\right)^{0.5} \left(1 - \left[1 - \left(\frac{\Theta - \Theta_{resid}}{n - \Theta_{resid}}\right)^{\frac{1}{m}}\right]^{m}\right)^{2}$$

m is function of pore size

Other Unsat. Flow Methods

- Kinematic Wave (assumes pressure gradients are negligible)
- Green-Ampt Method (infiltration as plug flow)