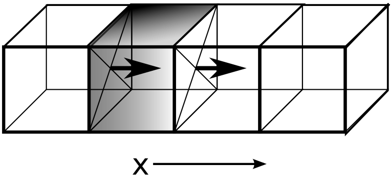
Conservation of Water

- assume coordinate system alligned with principle directions of anisotropy
- steady-state flow (no change in storage)
- calculate water balance for a cell
- To simplify derivation, limit flow to X direction



Conservation of Water

$$Q_x = -K_x A_x \frac{\partial h}{\partial x}$$

$$Q_{x(front)} - Q_{x(back)} \pm W = 0$$

$$(-K_x A_x \frac{\partial h}{\partial x})_{front} - (-K_x A_x \frac{\partial h}{\partial x})_{back} \pm W = 0$$

Dividing through by cell volume:

$$\frac{(-K_x \frac{\partial h}{\partial x})_{front} - (-K_x \frac{\partial h}{\partial x})_{back}}{\Delta x} \pm \frac{W}{Volume} = 0$$

$$\frac{\partial}{\partial x}(K_x \frac{\partial h}{\partial x}) \pm \frac{-W}{Volume} = 0$$

positive W is outward flow, minus sign reverses this convention.

Conservation of Water

What's this equation good for?

- find simple analytic solutions
- basis for flow nets
- computer simulation

Can quantify steady-state groundwater flow

$$\frac{\partial}{\partial x}(K_x \frac{\partial h}{\partial x}) \pm \frac{W}{Volume} = 0$$

note missing minus sign, $W>0$ indicates inflow

Flow Nets

Flow nets are:

- two-dimensional graphics
- show distribution of hydraulic head and flow direction
- quantitative
- steady state
- homogeneous

$$\nabla^2 h = 0 \tag{1}$$

$$\nabla^2 h = \frac{-W}{K \cdot V} \tag{2}$$

$$\nabla^2 h = \frac{-W}{T \cdot (Area)} \tag{3}$$

Flow Nets: Boundary Conditions

Type 1 (Dirichlet) boundary fixes a value or potential at the boundary, such as constant hydraulic head.

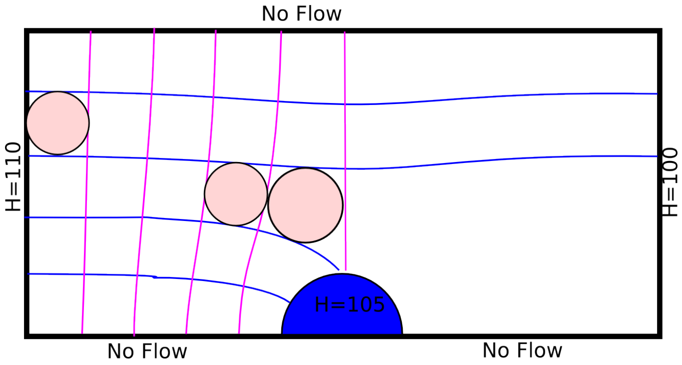
Type 2 (Neumann) boundary specify the flux normal to the boundary or gradient at a boundary, such as no flux across a boundary.

Type 3 (Cauchy) boundary relates the potential to the gradient or flux normal to the boundary. This is commonly called a hydraulic head dependent flux.

Creating a Flow Net

- assign boundary conditions
- scale properly (Vert. Exag., anisotropy)
- draw equipotentials and flow lines to form 'curvilinear' squares (inscribed circle must touch four sides)
- in homogeneous and isotropic setting:
  - equipotentials parallel constant head boundaries
  - streamlines at right angles to constant head boundaries
  - equipotentials at right angles to no flow boundaries
  - streamlines parallel to no flow boundaries

Discharge from Flow Nets



$$Q = \frac{K \cdot m \cdot H}{n}$$

Anisotropy in Flow Nets

- transform section so ellipse of K is a circle
- make flow net on transformed image
- 'un-transform' image along with equipotentials and streamlines

$$K = \sqrt{K_x \cdot K_y}$$

- equation based on stretching section to 'make isotropic'
- flow in un-stretched direction remains the same in both sections

$$Y^{stretch} = Y\sqrt{\frac{K_x}{K_y}}$$

$$q_x = K_x \cdot Y\frac{dh}{dx} = K_x^{strech} \cdot Y^{strech}\frac{dh}{dx}$$

$$K_x \cdot Y\frac{dh}{dx} = K_x^{strech} \cdot Y\sqrt{\frac{K_x}{K_y}}\frac{dh}{dx}$$

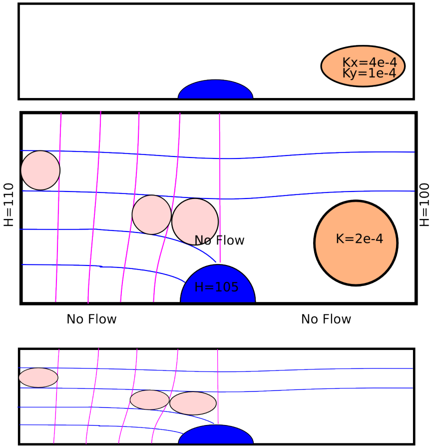
$$K_x = K_x^{strech}\sqrt{\frac{K_x}{K_y}}$$

$$K_x^{strech} = K_x\sqrt{\frac{K_y}{K_x}}$$

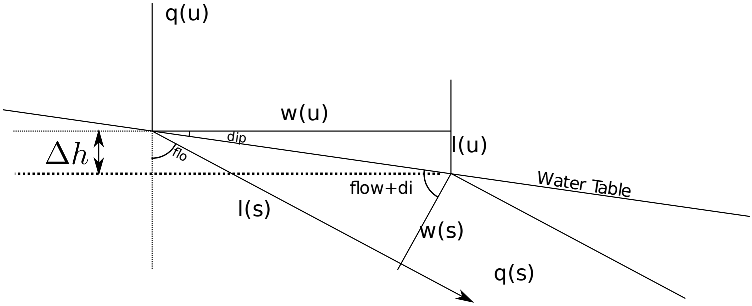
$$K_x^{strech} = \sqrt{\frac{K_y \cdot K_x^2}{K_x}}$$

$$K_x^{strech} = \sqrt{K_y \cdot K_x}$$

Anisotropy in Flow Nets



Including the Water Table



Assume flow in vadose zone is vertical:

$$q_s = K\frac{\Delta h}{\Delta l}$$

$$q_s = K\frac{\Delta h}{l_s}$$

$$q_s = K\frac{w_u \cdot tan(dip)}{l_s}$$

$$l_s = w_s \cdot tan(dip + flow)$$

$$q_s = Kw_s\frac{w_u \cdot tan(dip)}{w_s \cdot tan(dip + flow)}$$

$$Q = q_u \cdot w_u = q_s \cdot w_s$$

$$\frac{q_u \cdot w_u}{w_s} = K\frac{w_u \cdot tan(dip)}{w_s \cdot tan(dip + flow)}$$

$$q_u = K\frac{tan(dip)}{tan(dip + flow)}$$

Unconfined Flow

- Derivation of groundwater flow equation made an important assumption
  - Thickness of saturated zone constant
  - Not true in unconfined aquifers (but good approx. in some cases)
- If saturated thickness changes, cross-sectional area no longer constant

$$Q = -K(w \cdot h)\frac{dh}{dl}$$

- For horizontal flow:

$$\frac{\partial}{\partial x}\left(K\frac{\partial h^2}{\partial x}\right) + \frac{\partial}{\partial y}\left(K\frac{\partial h^2}{\partial y}\right) = W$$

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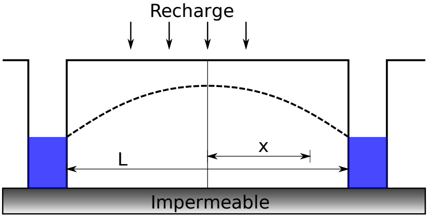
Simple Horizontal Flow

- Confined flow
  - Linear change in head for constant flux
  - $$q = K\frac{h_2 - h_1}{x_2 - x_1}$$
- Unconfined flow
  - Parabolic change in head for constant flux
  - Dupuit assumption
  - $$q = K\frac{h_2^2 - h_1^2}{2 \cdot (x_2 - x_1)}$$

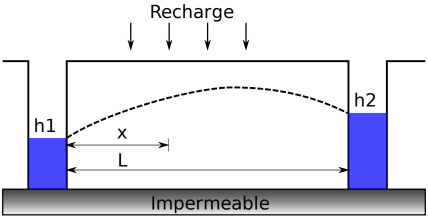
1-D Analytic Solution, Unconfined with Recharge

- Two const. head boundaries (same h),
- Unconfined flow,
- Dupuit assumption,
- Recharge (W)
- $h_x^2 = h_b^2 + \frac{W}{K}((\frac{L}{2})^2 - x^2)$, L is distance between boundaries and x is distance from center



1-D Analytic Solution, Unconfined with Recharge

- Two const. head boundaries (different h),
- Unconfined flow,
- Dupuit assumption,
- Recharge (W)
- $h_x^2 = h_L^2 - \frac{x(h_2^2 - h_1^2)}{L} + \frac{W}{K}(L-x)x$, x is distance from h_1



Confined 2-D Flow

- Potentiometric surface is plane
- $h = A \cdot x + B \cdot y + C$
- solve for A,B, and C using three data points with x,y and h
  - set up matrix eqn and solve
  - set one point with x=0,y=0,h=C; then use substitution to solve for A and B.

Confined 2-D Flow with Recharge

- $h = -\frac{N}{2 \cdot T}\left(D \cdot x^2 + (1 - D) \cdot y^2\right) + C$
- C is constant determined from boundary conditions
- D (from 0 to 1) controls eccentricity of equipotentials

$$D = \frac{\frac{b}{a}^2}{1 - \frac{b}{a}^2}$$

- a and b are lengths of major and minor axes