Types of Storage

- amount of water released from porous media due to unit drop in hydraulic head
- Storativity (S)(unitless)
  - confined aquifer
  - based on unit area (column of aquifer)
- Specific Storativity (S_s) (L^{-1})
  - confined aquifer
  - based on unit volume
- Specific Yield (S_y) (unitless)
  - unconfined aquifer
  - based on unit area

Sources of stored water

- Where does water flowing into a well come from the instant pumping starts?
- water released from an aquifer can come from several sources:
  - drainage, Specific Yield
  - expansion or compression of the aquifer materials (α)
  - expansion of water (β)
- as well is pumped, water is replace by inflow and storage decreases in importance

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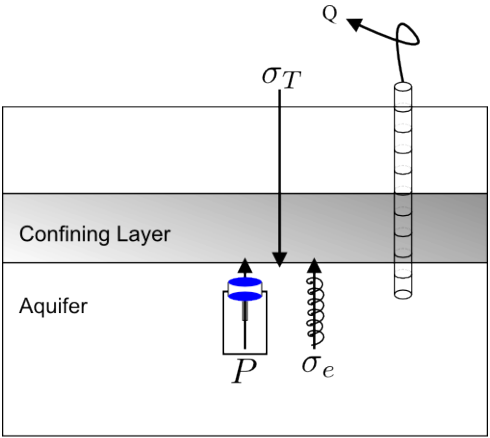


Figure: Effective stress and water pressure balance the total stress exerted on an aquifer.

If the total stress (σ_T) on an aquifer is constant then:

$$\Delta\sigma_e = -\Delta P = -\rho g \Delta h$$

If water is pumped into an aquifer, the water pressure increases and the effective stress on the aquifer media decreases.

$$\Delta V_{H_2O} \propto \Delta P$$
$$\Delta V_{H_2O} \propto -\Delta\sigma_e$$
$$\Delta V_{H_2O} = \Delta V_P - \Delta V_{aqfr}$$

aquifer and water compressibility

$$\beta = \frac{\frac{-\Delta V_{H_2O}}{V_{H_2O}}}{\Delta P} = \frac{\frac{\Delta \rho_{H_2O}}{\rho_{H_2O}}}{\Delta P}$$

$$\alpha = \frac{\frac{-\Delta V_{aqfr}}{V_{aqfr}}}{\Delta \sigma_e}$$

Combining these equations:

$$\Delta V_{H_2O} = (-\alpha V_{aqfr} \Delta \sigma_e) + (\beta V_{H_2O} \Delta P)$$

$$\frac{\Delta V_{H_2O}}{V_{aqfr}\Delta h} = \alpha\rho g + \beta\frac{V_{H_2O}}{V_{aqfr}}\rho g$$

$$\Delta\sigma_e = -\Delta P = -\rho g \Delta h$$

$$S_s = \rho g(\alpha + n\beta)$$

Specific Yield

- unconfined aquifer
- gravity drainage $>>$ expansion/compression of water/aquifer
- specific yield typically similar to porosity of aquifer material
- in some materials, water held in pore spaces not easily drained (eg. clay)

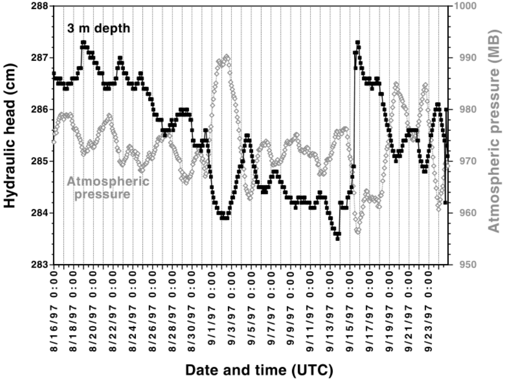
Implications of aquifer storage

- Tidal Oscillations
- Barometric oscillations

$$BE = \frac{\rho g \Delta h}{\Delta P_{atm}}$$

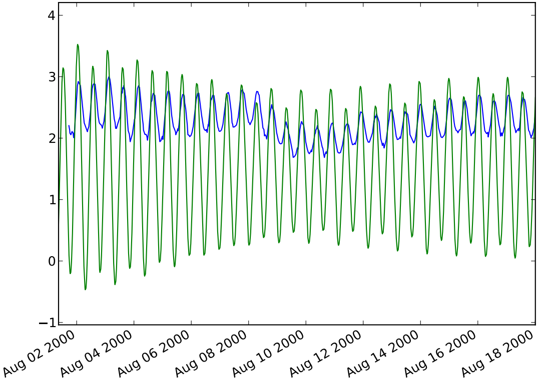
- landslides (decrease in σ_e)
- subsidence

Barometric Oscillations



$$BE = \frac{\rho g \Delta h}{\Delta P_{atm}} = \frac{n \cdot \beta}{\alpha + n \cdot \beta}$$

Tidal Oscillations



blue=Bayside well water level, green=Portland predicted tide level

Tidal Oscillations

- Tidal Efficiency
- $T.E. = \frac{\Delta h_{well}}{\Delta h_{ocean}} = \frac{\alpha}{\alpha + n \cdot \beta}$
- Usually can't measure true T.E., no well in ocean
- $Lag = x\sqrt{\frac{\pi \cdot S_s}{t_0 \cdot K}}$, $TE_{app} = TE_{true} exp\left(\frac{Lag \cdot 2\pi}{t_0}\right)$
- $S_s = \frac{n \cdot \beta \cdot \rho \cdot g}{1 - TE_{true}}$
- Need to convert ocean levels to fresh-water heads

t_0 is period of fluctuation

Carr and Van Der Kamp, 1969, Determining Aquifer characteristics by the tidal method. Water Resources Research 5:1023-1031.

Tully, NY Landslide

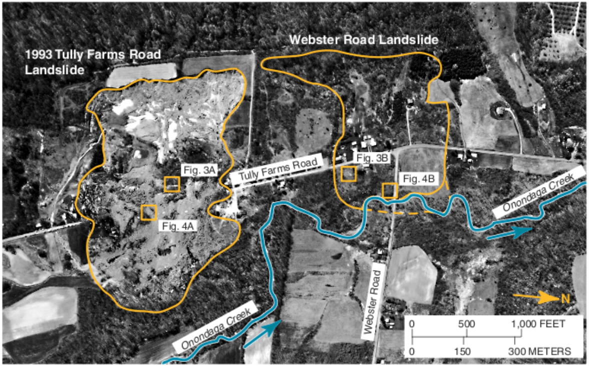


Figure 1. Aerial view of the Tully Farms Road landslide taken May 1, 1993, 4 days after the slide occurred and the approximate location of the Webster landslide, just to the north. Dashed line indicate probable extent of the Webster Road landslide beyond Onondaga Creek, and boxes indicate location of pictures shown in figures 3 and 4.

Tully, NY Landslide

- Heads in sand units higher than normal
  - Above normal rain
  - Thicker snow pack, more meltwater
- High pressure head exceeded total stress

USGS Fact Sheet 190-99. History of Landslides at the Base of Bare Mountain, Tully Valley, Onondaga County, New York

Storage and Groundwater Flow Equation

- Previously developed: $0 = \frac{\partial}{\partial x}\left(K \frac{\partial h}{\partial x}\right) \pm W$
- Assumed Steady State
- If Transient, need to consider storage
- Change in specific discharge in 1-D unit cell:

$$\frac{q_{front} - q_{back}}{\Delta x} = -S_s \frac{h(t_2) - h(t_1)}{\Delta t}$$

$$\frac{\partial}{\partial x}\left(K \frac{\partial h}{\partial x}\right) = S_s \frac{\partial h}{\partial t}$$

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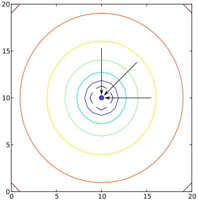
Flow to a Well

- When a well is pumped, initially have transient conditions
- Approaches steady-state conditions with time
- Pumping wells produce a radial flow pattern
- Radial flow in homogeneous, isotropic setting reduced to 1-D system



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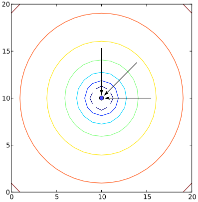
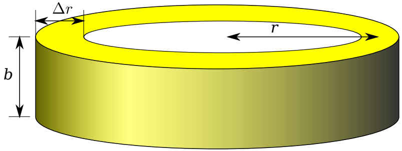
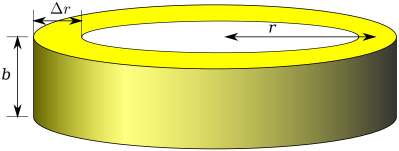


Radial Flow Equation



- Let Δr be small.
- Area at top of 'donut': $2 \cdot \pi \cdot r \cdot \Delta r$
- Area of outer edge of 'donut': $2 \cdot \pi \cdot r \cdot b$
- What is discharge across outer edge?
- $Q = -K \cdot 2 \cdot \pi \cdot r \cdot b \frac{dh}{dr}$

Radial Flow Equation



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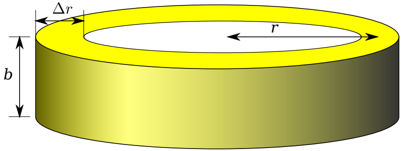

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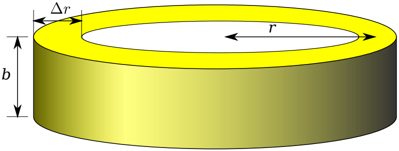
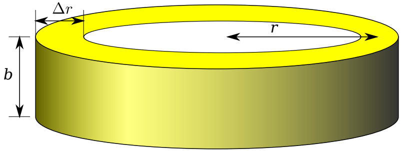


- What is radial groundwater flow equation?
- Set up mass balance $Q_{front} - Q_{back} = \Delta Storage$

Radial Flow Equation



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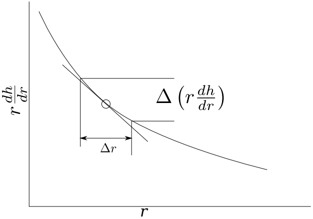
Radial Flow Equation



$$\frac{\Delta Q}{\Delta r} = 0$$

$$\frac{\Delta(2Kbr\frac{dh}{dr})}{\Delta r} = 0$$

$$\frac{d}{dr}\left(2Kb\frac{rdh}{dr}\right) = 0$$

- Assume steady state
- Based on graph, simplify diff. in fluxes to diff. eq.
- Apply product rule, and simplify.

Radial Flow Equation

- Add storage term
- General groundwater flow in radial coordinates:

$$S_s \frac{\partial h}{\partial t} = K \cdot \frac{\partial^2 h}{\partial r^2} + K \cdot \frac{1}{r} \frac{\partial h}{\partial r}$$

Radial Flow Equation

- Add storage term
- General groundwater flow in radial coordinates:

$$S_s \frac{\partial h}{\partial t} = K \cdot \frac{\partial^2 h}{\partial r^2} + K \cdot \frac{1}{r} \frac{\partial h}{\partial r}$$

Thiem Equation

- Solve steady state flow eqn.: $Q = 2 \cdot \pi \cdot r \cdot b \cdot K \frac{dh}{dr}$
- $h_2 - h_1 = \frac{Q}{2\pi T} \ln\left(\frac{r_2}{r_1}\right)$
- $H_2 = \frac{Q}{2\pi T} \ln(r_2) + C$ (if r_1 and h_1 fixed)

Superposition

- Can add solutions to linear diff. eq. together
- If have two pumping wells, add two thiem eqns. together
- $h = \frac{Q_1}{2\pi T} \ln(r_1) + \frac{Q_2}{2\pi T} \ln(r_2) + C$
- Can combine with other solutions, pumping well in regional gradient
- $h = \frac{Q_1}{2\pi T} \ln(r_1) + \frac{dh}{dx} \cdot x + \frac{dh}{dy} \cdot y + C$

Unconfined flow to a Well

- Not linear (no superposition)
- $h_2^2 - h_1^2 = \frac{Q}{\pi K} \ln\left(\frac{r_2}{r_1}\right)$
- with recharge:
- $h_2^2 - h_1^2 = \frac{Q}{\pi K} \ln\left(\frac{r_2}{r_1}\right) + \frac{W}{2 \cdot K}\left(r_1^2 - r_2^2\right)$

Transient Flow to a Well

- $S_s \frac{\partial h}{\partial t} = K \cdot \frac{\partial^2 h}{\partial r^2} + K \cdot \frac{1}{r} \frac{\partial h}{\partial r}$
- Solve based on many assumption:
  - Homogeneous, Isotropic aquifer
  - Infinite extent
  - No recharge
  - Constant pumping rate
  - Fully penetrating well
  - Confined aquifer

$$h_{t=0} - h_{r,t} = \frac{Q}{4\pi T} \int_u^\infty \frac{e^{-u}}{u} du = \frac{Q}{4\pi T} W(u)$$

$$u = \frac{r^2 S}{4Tt}$$

Transient Flow to a Well

finding W(u)

- look-up table
- $W(u) = -.5772 - ln(u) + u - \frac{u^2}{2 \cdot 2!} + \frac{u^3}{3 \cdot 3!} \cdots$
- Analytic solution (Srivastava and Guzman-Guzman, 1998, Ground Water (36)844)
  - if $u \geqslant 1$:
    - $a1 = u + 0.3575$
    - $a2 = (u \cdot exp(u))(u + 1.280)$
    - $W(u) = \frac{a1}{a2}$
  - otherwise:
    - $C1 = exp(-.577216)$
    - $W = ln(\frac{C1}{u}) + 0.9653 \cdot u - 0.1690 \cdot u^2$
- Numerical integration